Project #2: Chalice Design

Table of Contents:

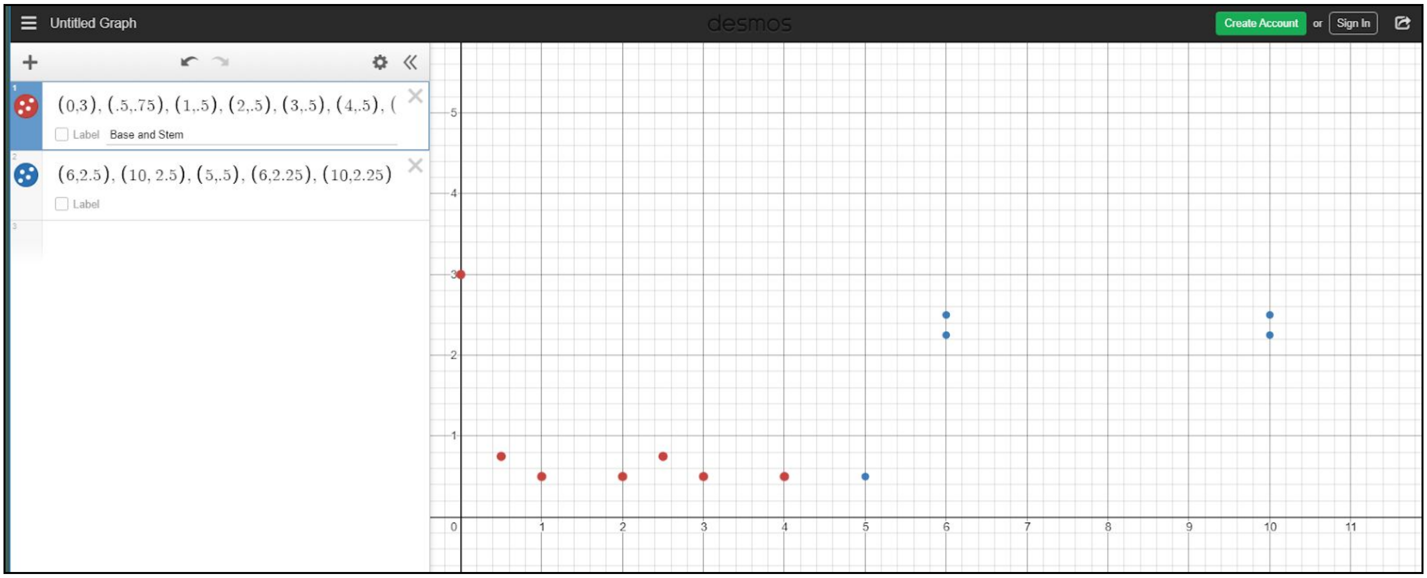
Introduction:

The time has come for the annual archery tournament. Wherever there is a tournament, there must be prizes! The royal court has selected a plethora of artisans to submit designs for the grand prize: the royal chalice.

Here is the work of one so-called artisan, may it please thee.

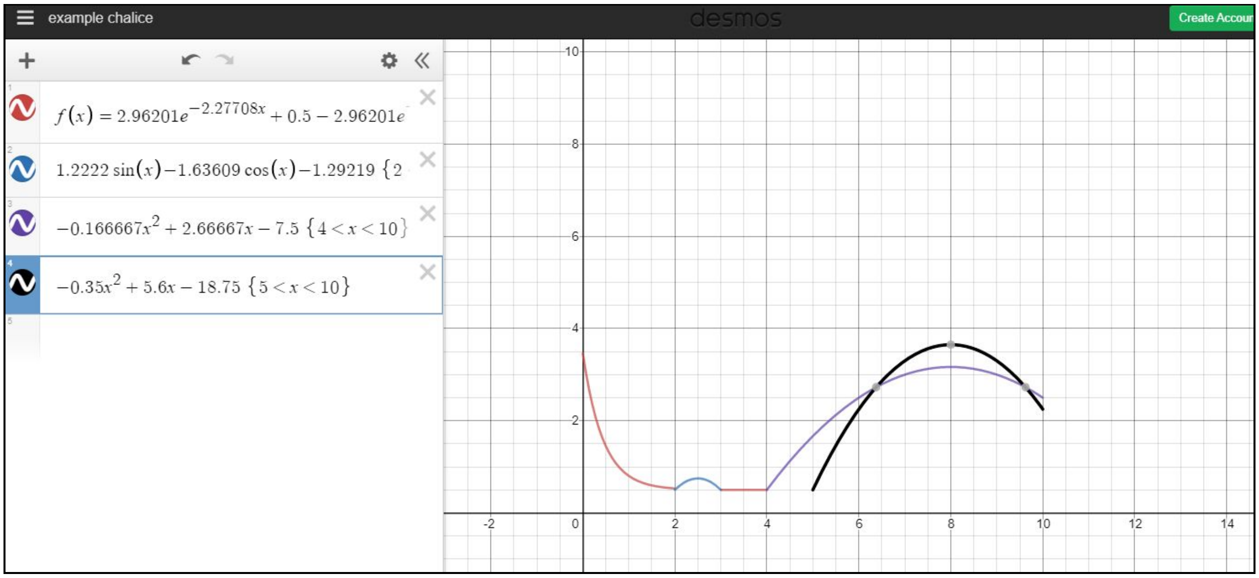
Part 1:

First, I begin plotting points on Desmos.com to start visualizing the chalice.
Caption: Initial points for chalice design. The base and stem are in red and the bowl is in blue.



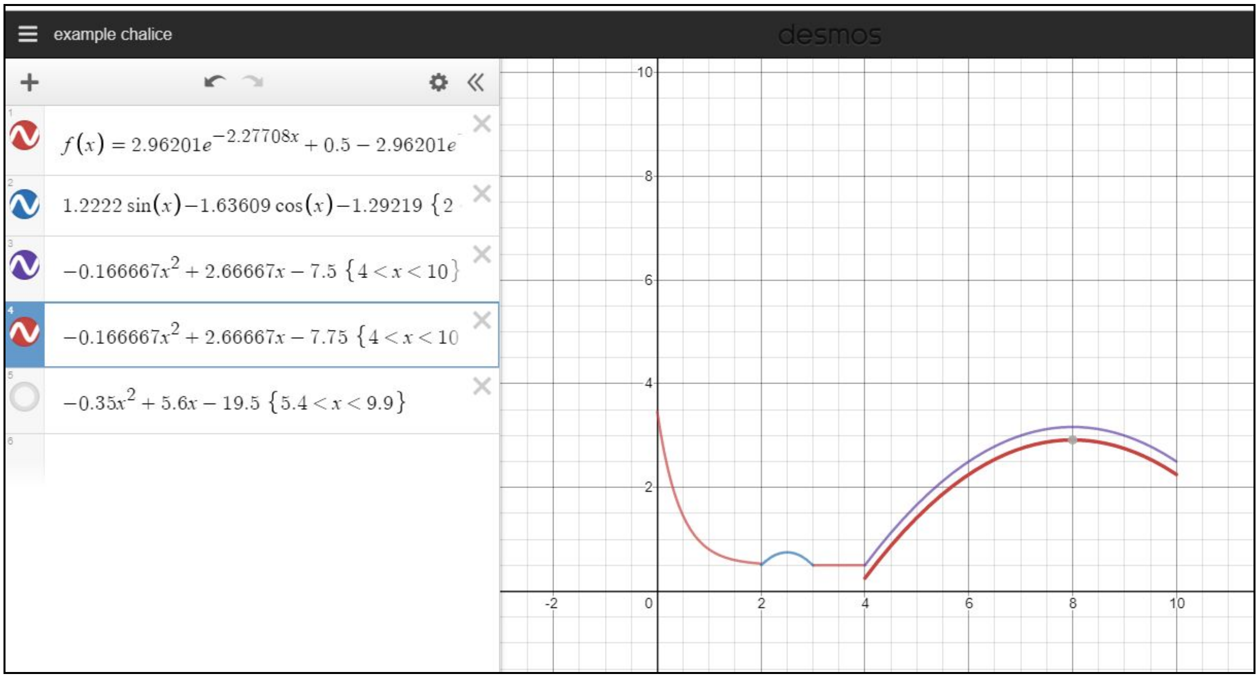
Then, I go through iterations of design, testing equations to meet the general design desired and to meet the guidelines of the competition (the equations are generated on WolframAlpha, see the Appendix).

First attempt at plotting functions using Desmos:



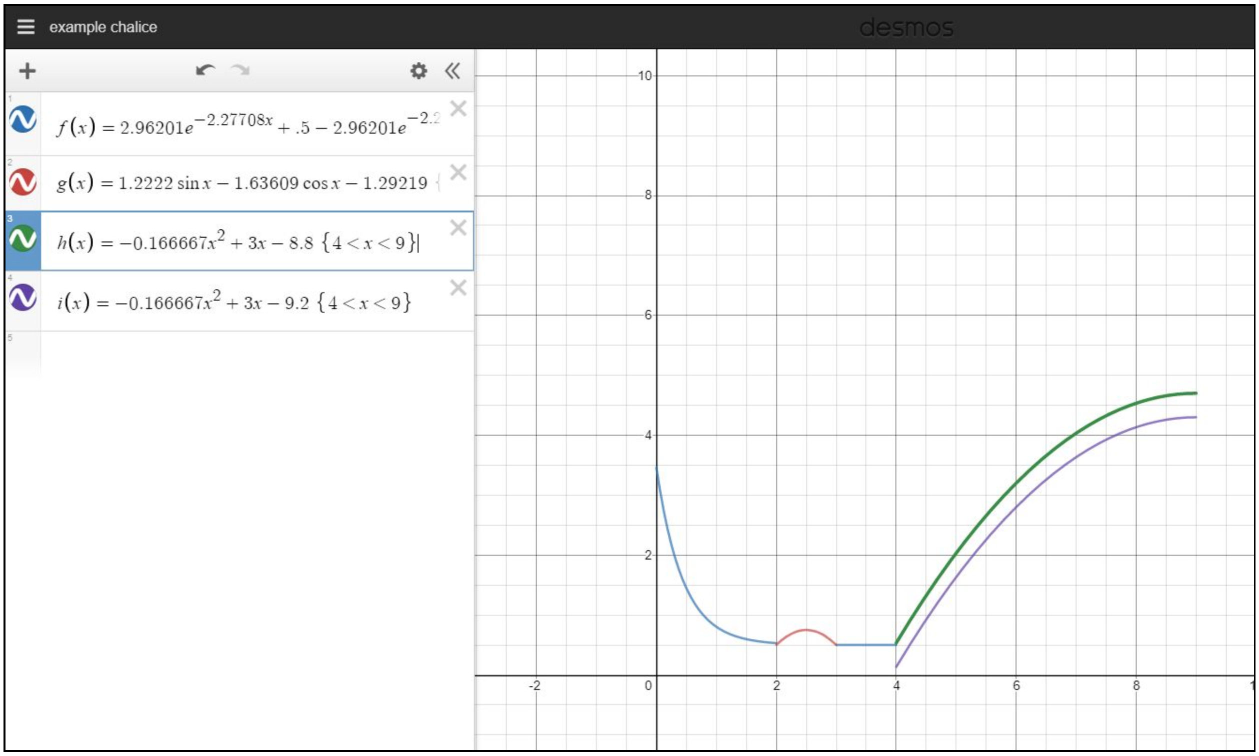
The first attempt is merely me taking the equations generated on WolframAlpha (an exponential fit, periodic fit, and two polynomial interpolations) and plugging them into Desmos for the desired domains of x. As you can see, it is a messy first attempt, but I do like the stem. It is the bowl that needs work.

Second attempt at plotting functions using Desmos:



The second attempt involves taking the formula for the outer portion of the bowl and shifting a copy of it down .25 centimeters for a formula for the inner portion of the bowl. This looks alright, but doesn't hold the necessary 150 cubic centimeters of wassail (aside: Wassail has Norse and Old English roots. You can read about the delicious drink on Wikipedia. A good-looking recipe is available at A Spicy Perspective).

Third attempt at plotting functions using Desmos.

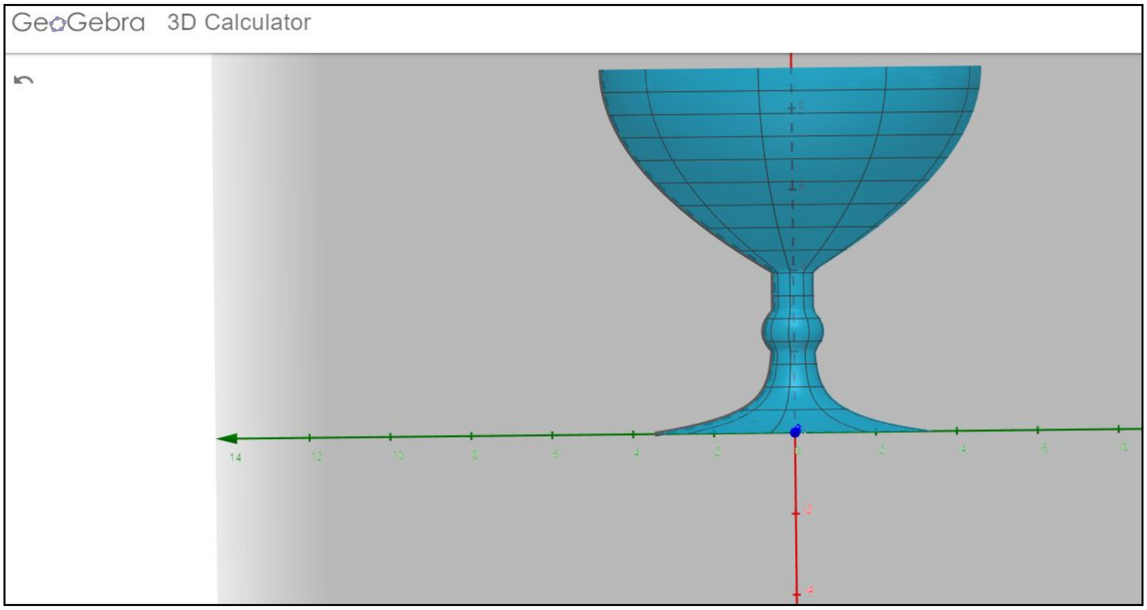


The third attempt adjusts the equations for the bowl so that it holds the necessary capacity. The bowl looks huge compared to the stem and base, but the chalice meets requirements for dishwasher safety (based on center of mass). I like to think of the chalice as miniature and meant to be held with two fingers.

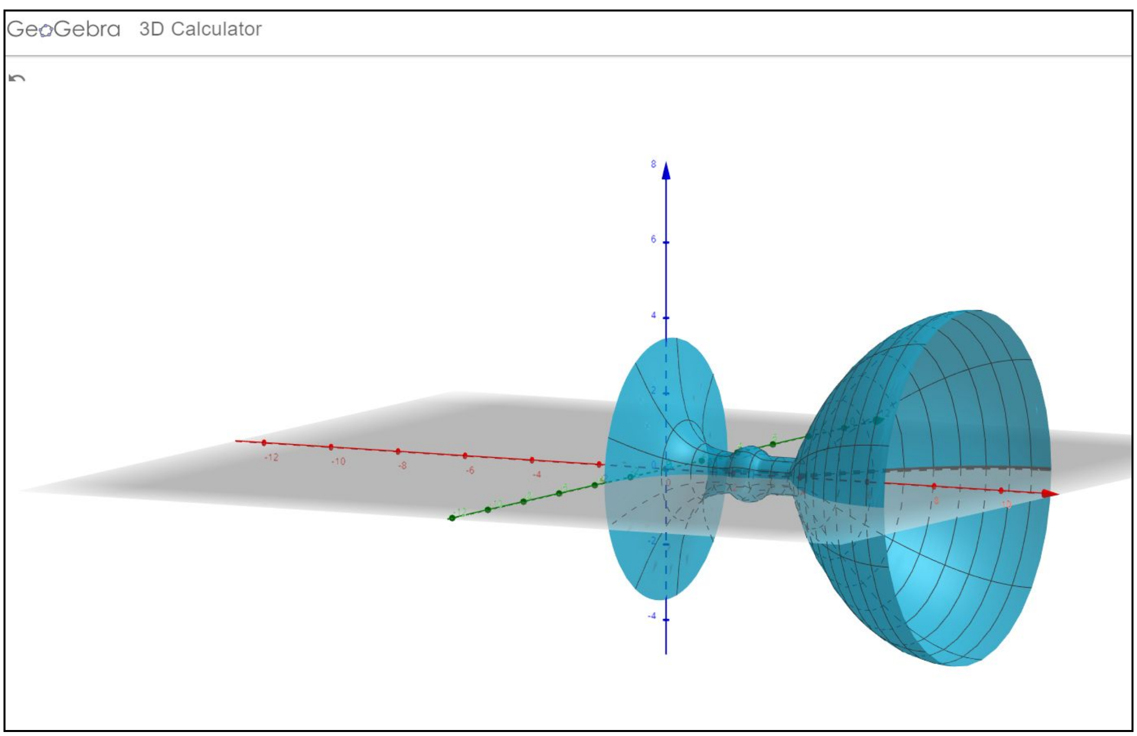
3-D Pictures of Chalice using Geogebra:

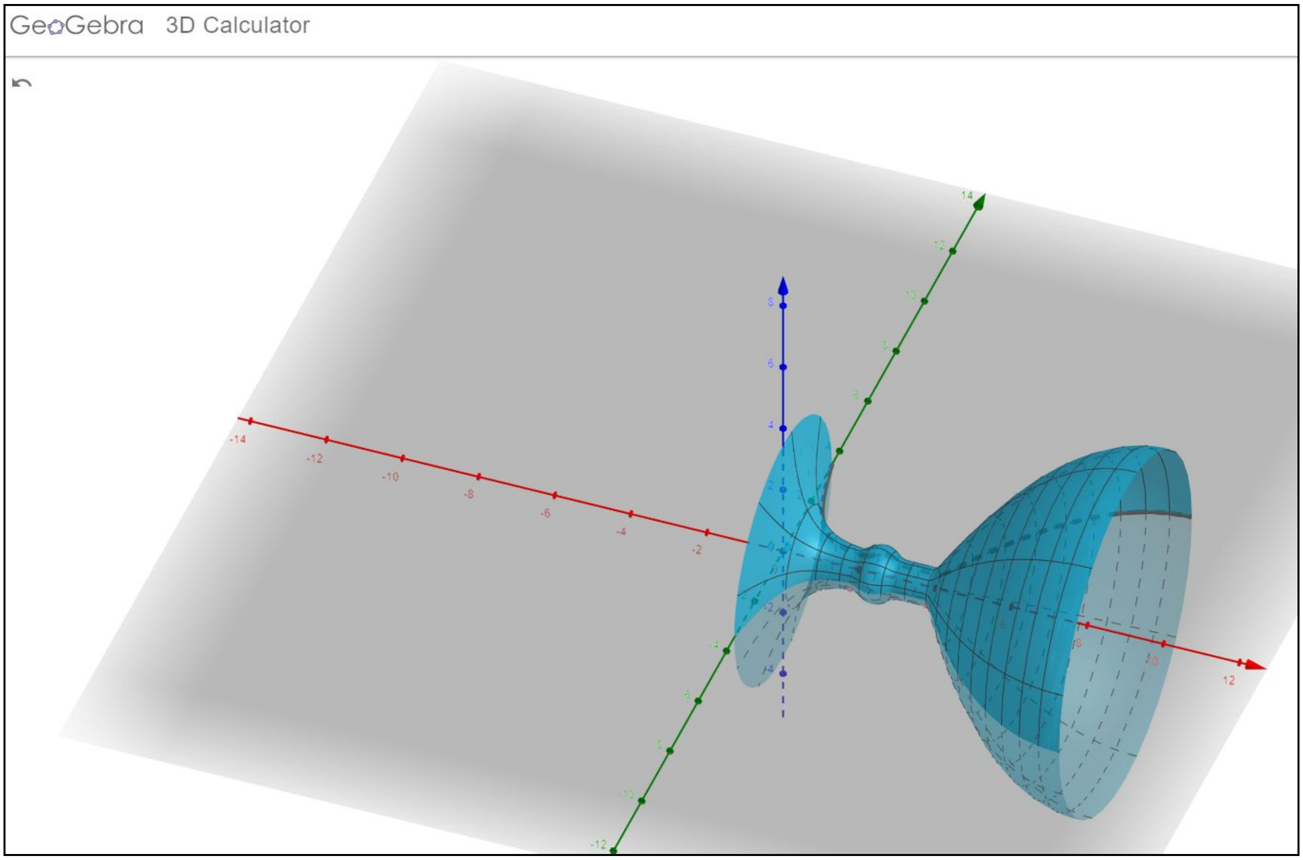
Finally, the time has come to look at the 3-d version of this chalice.

First, the standard profile:



Second, a couple of area views:





Part 2:

This part explains the requirements of the chalice-making competition and how my design meets these requirements. Frequent references will be made to the pages of the Appendix where exact calculations may be found. The requirements are,
“1. It must be a solid of revolution.
2. It must hold at least 150cm^3 of liquid.
3. It must use at most 150cm^3 of material to make.
4. The radius of the stem must be at least 0.5cm at its thinnest point.
5. The ratio of the height of its center of mass to the radius of the base must be at most 3:1” (copied from Moodle for easy reference).

1. The chalice is quite clearly a solid of revolution, as shown by the succession of images of in Part 1. If, however, the reader desires to see the integral that takes the piecewise function and revolves it around the x-axis to create a solid of revolution consult the section “Finding the volume of materials used in the chalice” in the Appendix.

2. The exact calculation can be found in the section “Integral for the volume/capacity of the bowl (using i(x))” of the Appendix.

   The capacity of the bowl is found by integrating i(x), the equation for the inner-portion of the bowl. The capacity of the bowl equals 157.357 cm^3, thereby exceeding the minimum 150 cm^3 capacity.

3. The exact calculation can be found in the section “Finding the volume of materials used in the chalice” in the Appendix.

   To calculate the volume of materials used, each section of the piecewise function must be revolved around the x-axis and the volume of that section calculated. The bowl is the one exclusion since the bowl is empty on the inside, but has a small volume along its edge (the volume between the equations h(x) and i(x).
   The volume of the chalice is, V(x) = 52.955641 cm^3, which is significantly less than the maximum volume of 150 cm^3.

4. The exact radius is not calculated at every point in the stem of the chalice, but the reader may consult the section “Exponential fit for part of the handle using WolframAlpha.”

   The data points that are given for the stem of the chalice in Part 1 maintain a minimum distance of 0.5 cm above the x-axis. The exponential fit f(x) = 2.96201e^(-2.27708x) does not maintain a minimum distance of 0.5 cm above the x-axis, but the equation f(x) was modified to maintain this minimum distance.

   The modified version, which is the version seen throughout Part 1, is f(x) = 2.96201e^(-2.27708x) + 0.5 - 2.96201e^(-2.27708*4). This maintains a distance of 0.5 cm above the x-axis, ensuring that the radius of the stem of the chalice is at least 0.5 cm thick at all times (and therefore dishwasher safe!).

5. The exact calculations are available in the section “Calculating the center of mass of the chalice” near the end of the Appendix.

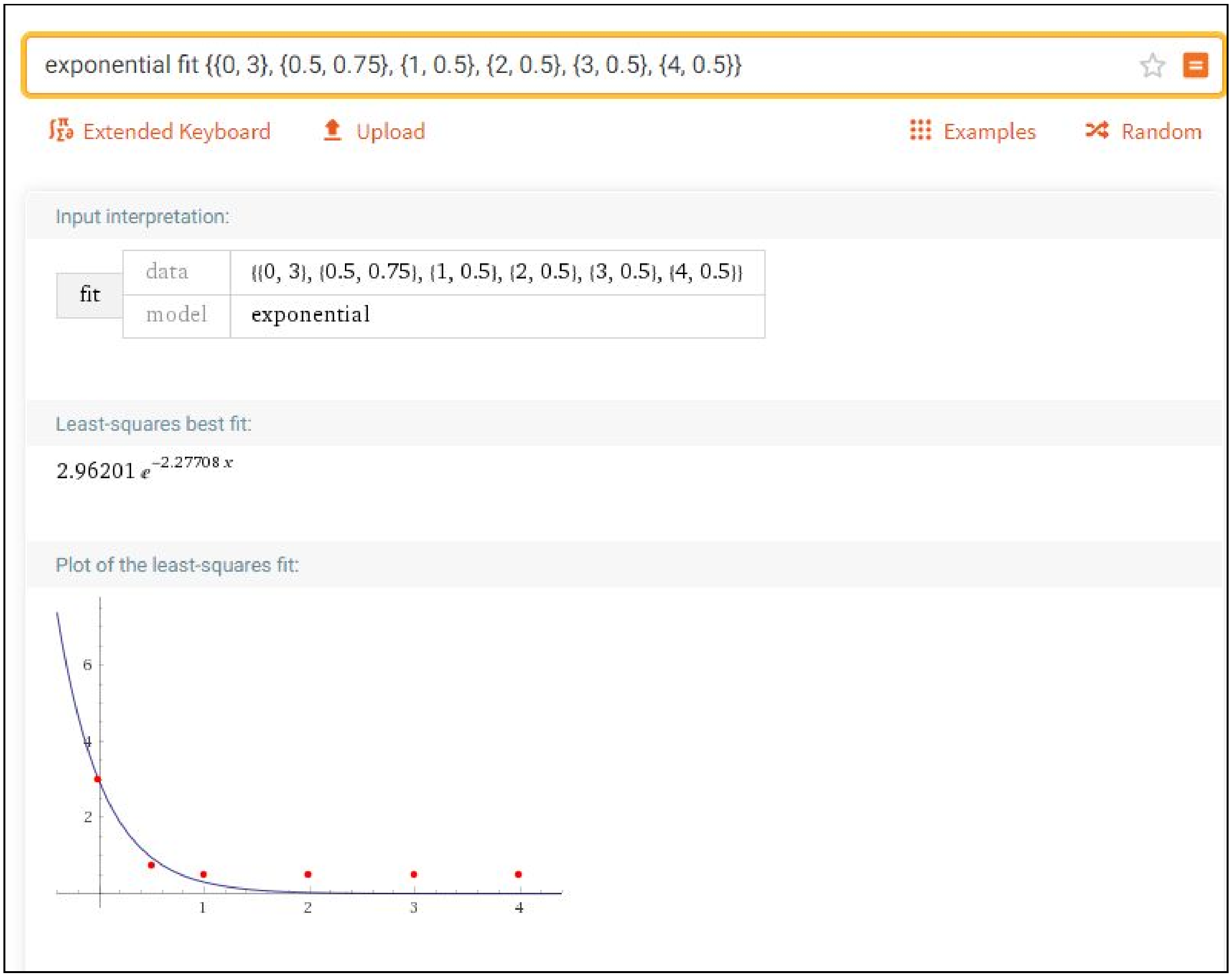
   x = 5.41596 is the height of the center of mass. The radius of the base is 3.The height of the center of mass x = 5.41596 is not more than 3 times the radius of the base, so the chalice meets the requirement and is stable.

This concludes Part 2, concerning the meeting of all requirements of my chalice. Knowing that the chalice meets the standards of the royal court, it is my humble desire that my chalice is selected as one worthy of being used as the archery tournament’s grand prize.

Appendix:

Here are data fits, interpolations, calculations, and other resources that may be useful in understanding and replicating this project.

Exponential fit for part of the handle using WolframAlpha:



This exponential fit does not maintain a distance of .5 cm above the x-axis at all times, so it must be adjusted to meet the requirements of the competition.

The equation used in the chalice design is the modified version of this exponential fit and is,

f(x) = 2.96201e^(-2.27708x)+0.5-2.96201e^(-2.27708*4).

This adjustment forces the exponential fit to go through the point (4, 0.5), thereby keeping the minimum distance of 0.5 cm above the x-axis at all times.

Periodic fit for part of the handle using WolframAlpha:

WolframAlpha computational intelligence.

periodic fit (2,.5) (2.5,.75) (3,.5)

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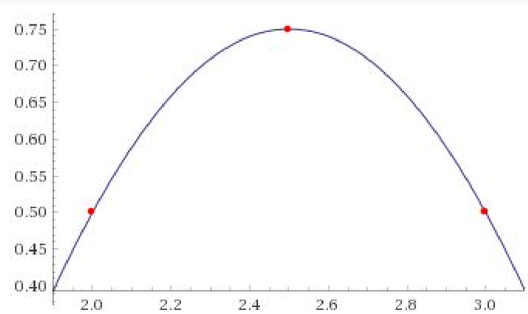
Input interpretation:

fit	data	{{2, 0.5}, {2.5, 0.75}, {3, 0.5}}
	model	periodic

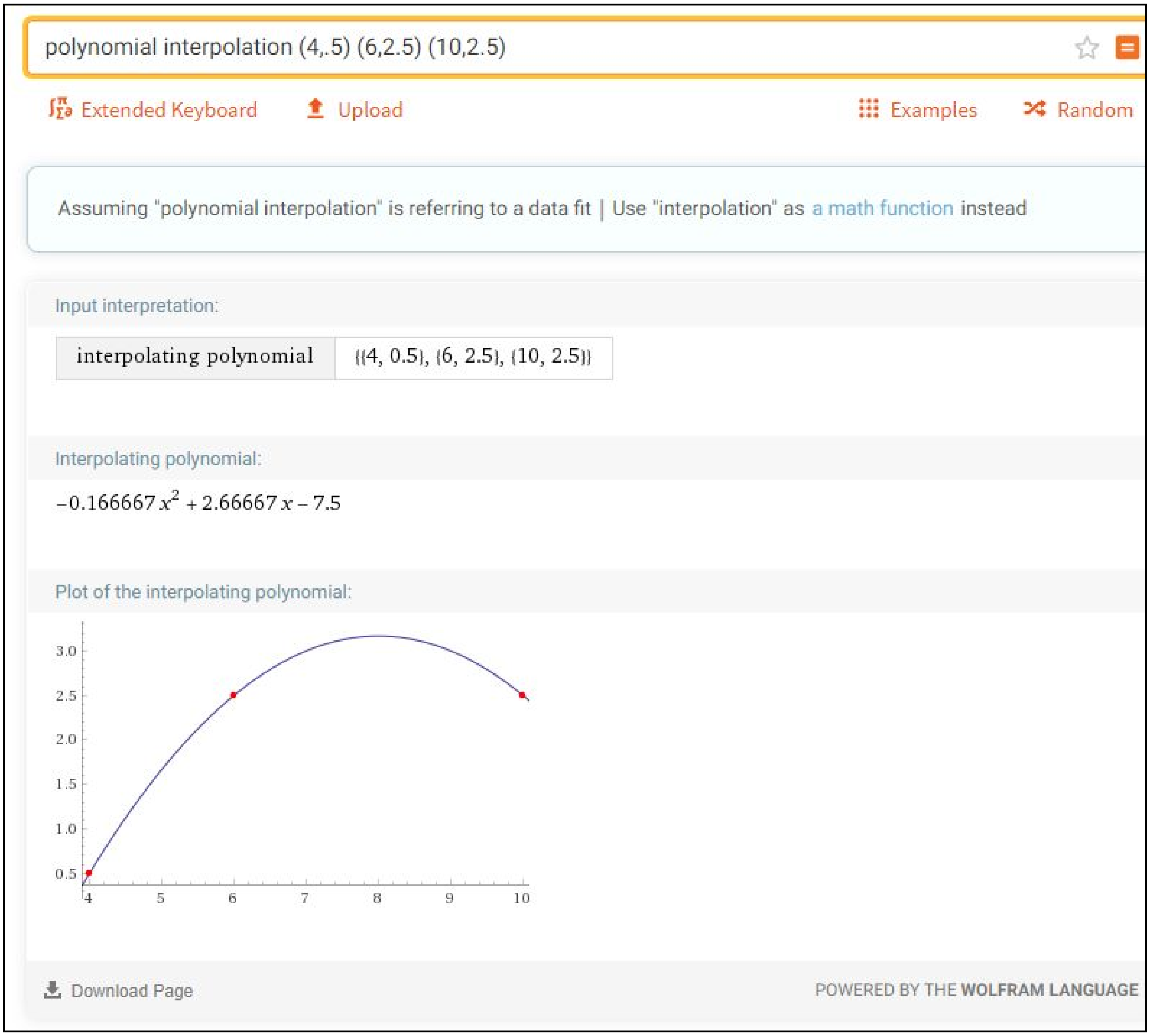
Least-squares best fit:

$1.2222\sin(x) - 1.63609\cos(x) - 1.29219$

Plot of the least-squares fit:



Polynomial interpolation for the outer-portion of the bowl using WolframAlpha:



Polynomial interpolation for the inner-portion of the bowl using WolframAlpha:

polynomial interpolation (5,.5) (6,2.25) (10,2.25)

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Assuming "polynomial interpolation" is referring to a data fit | Use "interpolation" as a math function instead

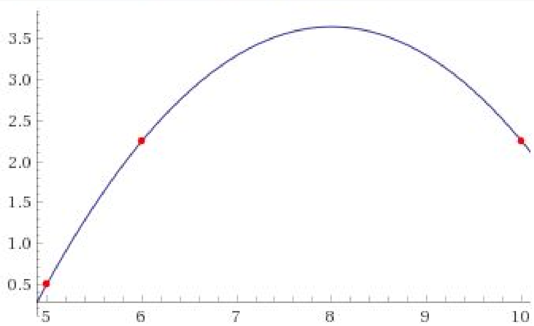
Input interpretation:

interpolating polynomial	{{5, 0.5}, {6, 2.25}, {10, 2.25}}

Interpolating polynomial:

$$-0.35\,x^2 + 5.6\,x - 18.75$$

Plot of the interpolating polynomial:



Finding the volume of materials used in the chalice:

Integral from 0 to 2 (using f(x)):

int ((pi*(2.96201e^(-2.27708x)+.5-2.96201e^(-2.27708(4)))^2), 0, 2, x)

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Definite integral: Step-by-step solution

$$\int_0^2 \pi (2.96201\, e^{-2.27708\, x} + 0.5 - 2.96201\, e^{-2.27708 \times 4})^2\, dx = 11.6612$$

Integral from 2 to 3 (using g(x)):

int ((pi*(1.2222sin(x) - 1.63609cos(x) - 1.29219)^2), 2, 3, x)

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Definite integral: Step-by-step solution

$$\int_2^3 \pi (1.2222 \sin(x) - 1.63609 \cos(x) - 1.29219)^2\, dx = 1.41091$$

Integral from 3 to 4 (using f(x)):

int ((pi*(2.96201e^(-2.27708x)+.5-2.96201e^(-2.27708(4)))^2), 3, 4, x)

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Definite integral: Step-by-step solution

$$\int_3^4 \pi (2.96201\, e^{-2.27708\, x} + 0.5 - 2.96201\, e^{-2.27708 \times 4})^2\, dx = 0.788331$$

Integral from 4 to 9 (using h(x) and i(x)):

int ((pi*(-0.166667x^(2)+3x-8.8)^2)-(pi*(-0.166667x^(2)+3x-9.2)^2), 4, 9, x)

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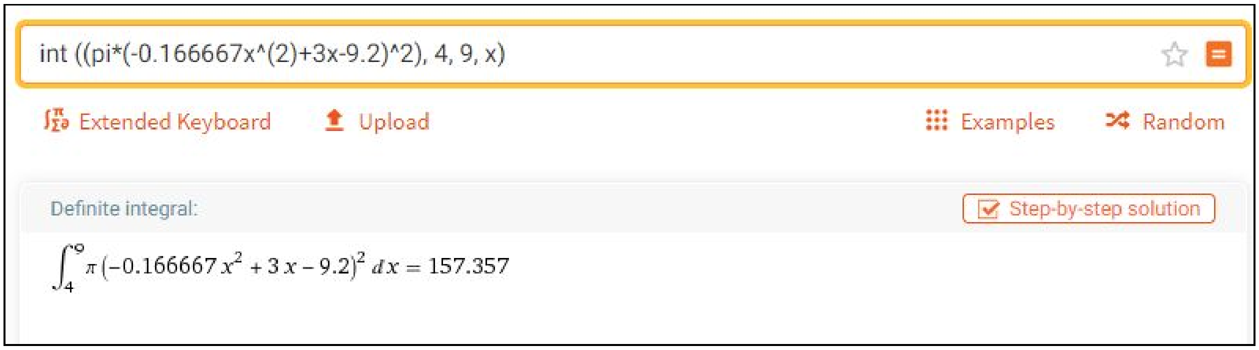
Definite integral: Step-by-step solution

$$\int_4^9 (\pi (-0.166667\, x^2 + 3\, x - 8.8)^2 - \pi (-0.166667\, x^2 + 3\, x - 9.2)^2)\, dx = 39.0952$$

The volume of the chalice is, V(x) = 52.955641 cm^3.

Integral for the volume/capacity of the bowl (using i(x)):



Calculating the center of mass of the chalice:

The equation is int(x*(outer^2 - inner^2)dx)/ int(outer^2 - inner^2 dx)

For the numerator:
int((x*(2.96201e^(-2.27708x)+.5-2.96201e^(-2.27708(4)))^2), 0, 2, x) +
int((x*(1.2222sin(x)-1.63609cos(x)-1.29219)^2), 2, 3, x) +
int((x*(2.96201e^(-2.27708x)+.5-2.96201e^(-2.27708(4)))^2), 3, 4, x) +
int(x*((-0.166667x^2+3x-8.8)^2-(-0.166667x^2+3x-9.2)^2), 4, 9, x)

OR

integral_0^2 x (2.96201 e^(-2.27708 x) + 0.5 - 2.96201 e^(-2.27708 4))^2 dx **= 1.4594**
integral_2^3 x (1.2222 sin(x) - 1.63609 cos(x) - 1.29219)^2 dx **= 1.12277**
integral_3^4 x (2.96201 e^(-2.27708 x) + 0.5 - 2.96201 e^(-2.27708 4))^2 dx **= 0.878047**
integral_4^9 x ((-0.166667 x^2 + 3 x - 8.8)^2 - (-0.166667 x^2 + 3 x - 9.2)^2) dx **= 87.8329**
SUM **= 91.293117**

For the denominator:
int((2.96201e^(-2.27708x)+.5-2.96201e^(-2.27708(4)))^2), 0, 2, x) +
int((1.2222sin(x)-1.63609cos(x)-1.29219)^2), 2, 3, x) +
int((2.96201e^(-2.27708x)+.5-2.96201e^(-2.27708(4)))^2), 3, 4, x) +
int((-0.166667x^2+3x-8.8)^2-(-0.166667x^2+3x-9.2)^2), 4, 9, x)

OR

integral_0^2 (2.96201 e^(-2.27708 x) + 0.5 - 2.96201 e^(-2.27708 4))^2 dx **= 3.71187**
integral_2^3 (1.2222 sin(x) - 1.63609 cos(x) - 1.29219)^2 dx **= 0.449107**
integral_3^4 (2.96201 e^(-2.27708 x) + 0.5 - 2.96201 e^(-2.27708 4))^2 dx **= 0.250934**
integral_4^9 ((-0.166667 x^2 + 3 x - 8.8)^2 - (-0.166667 x^2 + 3 x - 9.2)^2) dx **= 12.4444**
SUM **= 16.856311**

91.293117/16.856311 **= 5.41596 = x** is the height of the center of mass.

The radius of the base is 3.The height of the center of mass x = 5.41596 is not more than 3 times the radius of the base, so the chalice is stable and able to stand.

Recreating Results: Points and Equations Used

1. If you want to plot the points I began with, go to Desmos.com and copy/paste the following points:

   ```
   (0,3),(.5,.75),(1,.5),(2,.5),(3,.5),(4,.5),(2.5,.75),(6,2.5),(10, 2.5),(5,.5),(6,2.25),(10,2.25)
   ```

2. If you want to recreate my third and final design, go to Desmos.com and copy/paste the following equations:

   ```
   f\left(x\right)=2.96201e^{-2.27708x}+.5-2.96201e^{-2.27708\left(4\right)}\ \left\{0<x<2,\
   3<x<4\right\}
   g\left(x\right)=1.2222\sin x-1.63609\cos x-1.29219\ \left\{2<x<3\right\}
   h\left(x\right)=-0.166667x^{2}+3x-8.8\ \left\{4<x<9\right\}
   i\left(x\right)=-0.166667x^{2}+3x-9.2\ \left\{4<x<9\right\}
   ```

3. If you want to recreate the 3-d model, go to Geogebra.org/3d?lang=en and copy/paste the following,
   a. This first equation sets the piecewise function for my chalice specifically,

      ```
      f(x)=If(x>0∧x<2,2.96201e^(-2.27708x)+0.5-2.96201e^(-2.27708*4),If(x>2∧x<3,1
      .2222sin(x)-1.63609cos(x)-1.29219,If(x>3∧x<4,2.96201e^(-2.27708x)+0.5-2.962
      01e^(-2.27708*4),If(x>4∧x<9,-0.16667x^(2)+3x-8.8))))
      ```

   b. This second equation rotates the piecewise function about the x-axis to create the 3-d model:

      ```
      a=Surface(t,f(t)cos(u),f(t)sin(u),t,0,9,u,0,2π)
      ```